Introduction to Fractal Geometry

Martin Churchill, 2004

1. A few Random Quotes

Clouds are not spheres, coastlines are not circles, bark is not smooth, nor does lightning travel in straight lines.
Mandelbrot

"And are you not", said Fook leaning anxiously forward, "a greater analyst than the Googleplex Star Thinker in the Seventh Galaxy of Light and Ingenuity which can calculate the trajectory of every single dust particle throughout a five-week Dangrabad Beta sand blizzard?"
"A five-week sand blizzard?" said Deep Thought haughtily. "You ask this of me who have contemplated the very vectors of the atoms in the Big Bang itself? Molest me not with this pocket calculator stuff."
Douglas Adams

I wonder whether fractal images are not touching the very structure of our brains. Is there a clue in the infinitely regressing character of such images that illuminates our perception of art? Could it be that a fractal image is of such extraordinary richness, that it is bound to resonate with our neuronal circuits and stimulate the pleasure I infer we all feel?
P. W. Atkins

2. Introduction

Throughout recent years much development and interest has gone into concepts such as the newly rising Chaos theory. I'm going to discuss something that could possibly considered a specific part of this, something generally known as fractal geometry.

One definition of the subject I've found claims that fractal geometry can be considered different to classical geometry in that it does not deal in integer dimensions. However, this probably sounds mind-boggling, as a dimension is defined as the size of a basis that cannot, of course, have a non-integral cardinality. However, hopefully before the end of this document I'll be able to convince you that the idea of represent fractals as having fractional dimensions is a reasonable conceptualisation to make, and it is indeed possible to use it as a definition of a fractal (providing, of course, you use a different definition of 'dimension.' More on this later.)

Before I begin going into the mathematics of how fractals work, I ask you this question: What is the length of the coastline of Britain? Immediately one could go out, ignoring of course all of the practical problems involved, and wander around Britain with their trusty 15-inch rule and spirit level and a few years later get back with a reasonable result. However, one may argue the 15-inch rule is clearly a straight rigid object: it would have been able to measure all of the inny-and-outy-bits that are on a smaller scale than 15 inches – the resolution of the journey, if you like, is 15 inches. So one can get a tape measure and try it again, but still that's limiting to the thickness of the tape measure. One can go further and further down, and on each measurement the perimeter measured would grow and grow, limitlessly. So there is no sensible answer to that question. Now one could argue that there does indeed exist a fundamental level of fundamental atoms, or that space is discrete, therefore it is possible to define. However this clearly cannot work assuming space is continuous. It wouldn't be hard to argue, from the analogy above, that the coast of Britain has a finite area but an infinite perimeter. Quite a strange concept at first.

So, perhaps it is these continuous spaces that allow these strange things like the above described to exist, and anyway the above is all very hand-wavey, so of course the natural thing for us to do is to be more explicit and try to find things like the above phenomena in nice well-defined places like the complex plane. Consider the function defined by

$$z_0=z \; ; \; z_{n+1} = z_n^2 + z$$

Now, let M be the set $\{z \in C$ such that z_n does not diverge to infinity$\}$, called the Mandelbrot set. This is really quite simple to define – after all, it should be reasonable easy to determine which values of c cause a sequence that tends to infinity and which do not, obviously when you get sufficiently big c it will diverge to infinity, so it's just finding that limit. Well, yes, in a way. But the shape you get is quite interesting – one can 'zoom in' on the perimeter of this shape and it just never gets less complicated, patterns of the original shape appear on the edge of the main shape, and this continues forever, no matter how small the scale becomes: turtles all the way down.[1]

3. *Types of fractals*

It is possible to characterise fractals into different 'types' – a couple that we shall have a look at include self-similar fractals and self-repeating fractals. Of course some fractals, such as the aforesaid coast of Britain, may not have repeating patterns – they may be 'irrational' if you like – or, such as the Mandelbrot set, there might be patterns on a smaller level similar to patterns on a higher level, but not quite the same. I shall now look into fractals that replicate themselves directly, and the 'dimension' of these fractals.

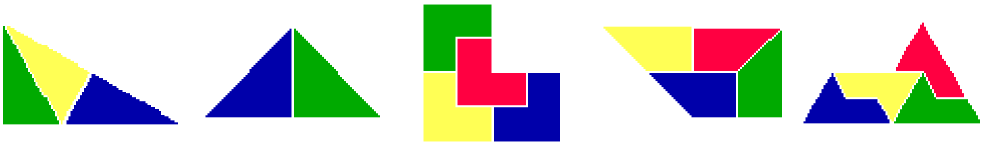
4. *Self-similar Fractals*

Many shapes, even non-fractals (i.e. those with an integer dimension, although we still haven't defined this) are self-similar. Let A and B be shapes. Then A is said to be *similar* to B if there is an isomorphism from A to B (i.e. if B can be obtained by a sequence (composition) of translations, rescalings and rotations of A.)

---

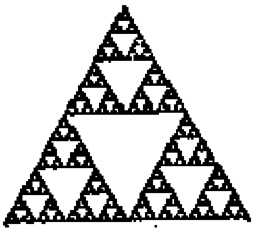
[1] A nice *Brief History of Time* reference.

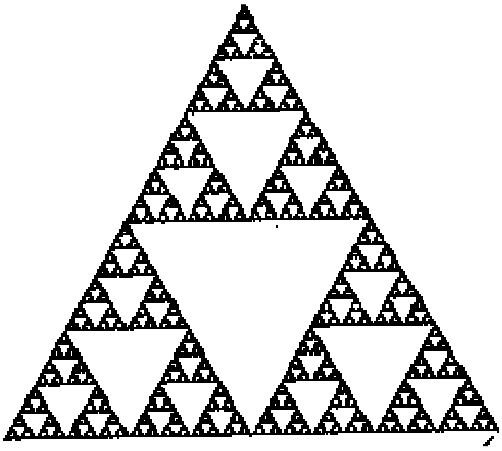
Behold:



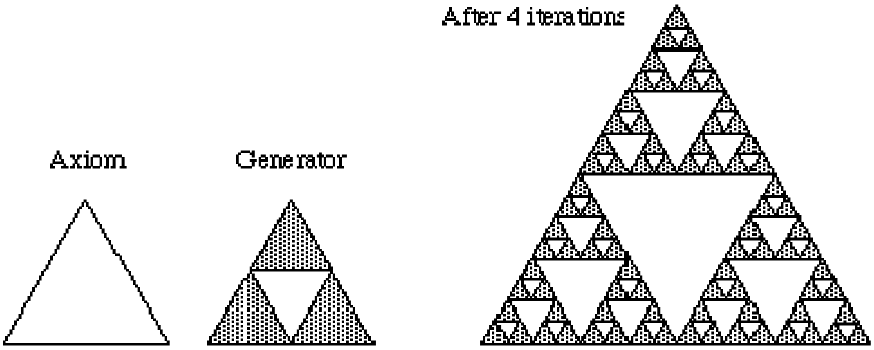
(Images: http://math.rice.edu/~lanius/fractals)

All of the above are self-similar, i.e. the shape entirely consists of 'joining' together shapes that are similar to the shape itself. Now consider the following shapes:





The second image can be observed as being a larger version of the first image (as if one zoomed in on the former image.) However it also appears to be three versions of the larger shape placed together: this is an example of a self-similar fractal. The shape above (known as the Sierpinski Gasket) is made by placing together three Sierpinski Gaskets, each of which consists of three Sierpiknski Gaskets… indeed turtles all the way down. It can also be made from a 'top-down' approach by considering an *axiom* (the initial shape) and its *generator* (a function between two shapes.) In the above case we have:

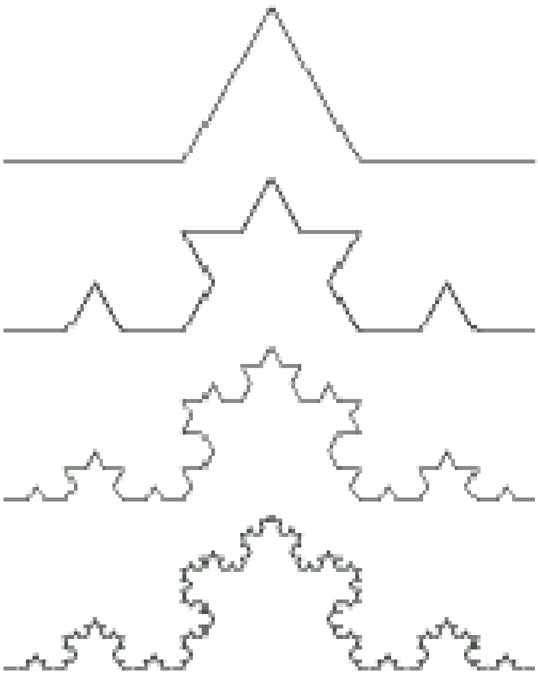


I.e. we take a sequence of shapes (a_n), with a_0 defined as the 'axiom' and a_n defined as the generator applied to a_{n-1} (in this case cutting out the middle triangle of any instances of the axiom within the shape.) The final fractal is then the limit of this sequence, however that is defined. This axiom-generator approach seems a nice, well-defined way of generating self-similar fractals.

(Images of the Sierpinksi Gasket from http://astronomy.swin.edu.au/~pbourke/fractals/gasket/)

5. Further Examples of the Axiom-Generator Concept

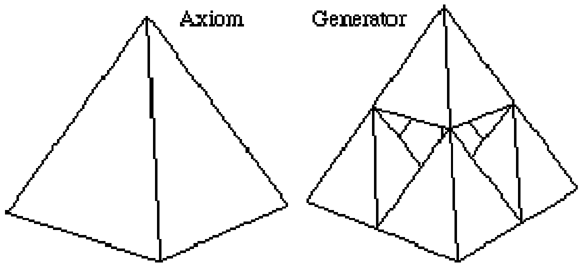
Other examples of self-similar fractals defined in this way include the Koch curve:

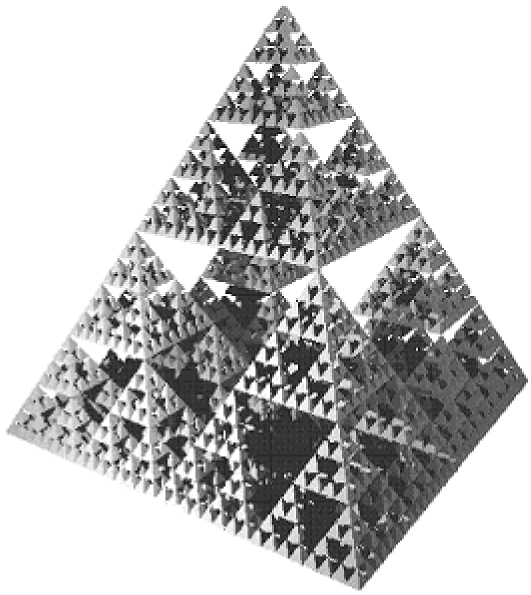


Here the axiom is first shape and the generator takes any straight line in the curve and replaces for a (rotated, rescaled[2]) version of the first shape here. The effect is shown on the left.

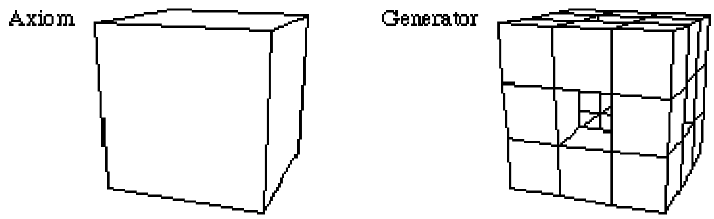
A three-dimensional version of the Sierpinski gasket:

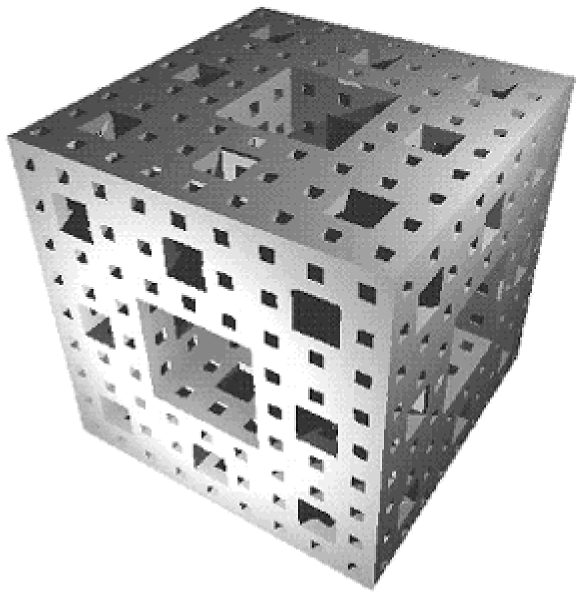
[2] I may be tempted to use the word 'isomorphed' here but I don't want to use too many nonstandard words…





And the Menger sponge:





6. Further Analysis of the Gasket

Let us consider a Sierpinksi Gasket whose axiom is a triangle, of unit area. It's clear that the first iteration removes a quarter of the area, the second iteration a further 3/16, the then 9/64. Define A_n as the area removed from the gasket in a_n and we have

$$A_n = \tfrac{1}{3}\sum_{i=1}^{n}\left(\tfrac{3}{4}\right)^i$$

Thus the area of the gasket itself is 0 as $A_n \to 1$ as $n \to \infty$ (using the geometric series formula, $\tfrac{1}{3}\sum_{i=1}^{n}\left(\tfrac{3}{4}\right)^i = \tfrac{1}{3}\left(\sum_{i=0}^{n}\left(\tfrac{3}{4}\right)^i - 1\right) = \tfrac{1}{3}\left(\frac{1}{1-\tfrac{1}{4}} - 1\right) = 1$) so the area of the gasket as $n \to \infty$ is zero. Let us now consider the area perimeter of a_n, P_n with initial perimeter 1. After the first iteration the perimeter is 1+1/2, after the second 1+1/2+3/4, giving

$$P_n = 1 + \tfrac{1}{3}\sum_{i=1}^{n}\left(\tfrac{3}{2}\right)^i$$

Now $P_n \to \infty$ as $n \to \infty$ (geometric series.) Thus the Sierpinski Gasket has an infinite perimeter but zero area[3] – an odd concept indeed. This relates to what I mentioned earlier, and identifies the Sierpinski Gasket in the same category as the coast of Britain as discussed above. Before long we shall explicitly define this category when we explore this concept of the 'fractal dimension' I have so teasingly hinted about before. However, firstly I shall explain an interesting place the Gasket can be found (apart from this document, which may be as argued as interesting depending on one's perspective, and, indeed, opinion.)

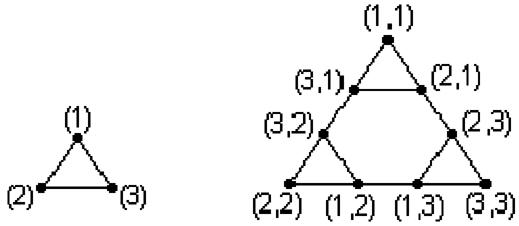
7. Interesting Application of the Gasket

Last year at a talk by Ian Stewart revealed an interesting application of the said Gasket in a link with the age-old puzzle the Tower of Hanoi.[4]

[3] Here I have chosen to assume that the perimeter of the limit of a_n is the limit of P_n and likewise with the area – i.e. the two limits are interchangable. However, it doesn't seem too unwise to say that area (perimeter) of the final gasket the limit of the area (perimeter) of the sequence by definition.

[4] The tower of Hanoi is a game with the set up as illustrated. Players can move discs from one peg to another as long as on any particular peg the discs are decreasing in size. All of the discs start on one peg (in decreasing order) and the game is won when all of the pegs are on a different peg (also in decreasing order – although the content of this bracket is deducible from previous facts.)

Here (a_n) represents the game with n different discs, each vertex of an representing a valid configuration of discs. For example, in one-disc and two-disc games we have:



Where n-tuples represent the position of the discs (1, 2 or 3 representing the different pegs) represented in increasing order. The only edges that are included are those of valid moves (by the Laws of the Game, on a specific peg the discs must be arranged in decreasing order of size.) The fact that this shape is generated is due to the fact that only valid n-tuples are included in the gasket, but it also suggests something slightly more cunning and subtle. The fact that the game can be represented by the Sierpinski Gasket seems to suggest that the game consisting of n discs can be solved by solving two versions of solving the game of n-1 discs (we want to go from one extreme vertex to another – the logical (and quickest) route being 'down one of the sides' of the triangle, which involves going from one extreme vertex to the other of two subgaskets of n-1 discs.) : This is true, to solve for 255 discs, move the first 254 discs from Peg 1 to Peg 2 (which we can do,) then move the remaining peg to Peg 3 and then the first 254 discs from Peg 2 to Peg 3. A nice recursive solution which defines how to solve the problem. (Each of the three subgaskets[5] refers to the puzzle with 254 disks with the 255^{th} disc residing in one of the three remaining pegs.) The gasket also has other applications: when the catering industry catch on, they could create a Sierpinski potato, one that is infinitely crispy but has zero calories.

8. Fractal Dimension

Here I shall attempt to define the *fractal dimension* of a shape (this is not the classical definition of a dimension, i.e. the cardinality of a basis, however these two values are equal when the fractal dimension is an integer. It is also known as the Hausdorff dimension.) This definition relates to the more intuitive definition of dimension for a geometrical shape, and is thus more specialised than the classical definition of dimension.

Consider a spherical cow, of radius R. If its radius is doubled to 2R, its mass[6] is increased by a ratio of 2^3. Similarly for a circular cow, its mass would be increased by a ratio of 2^2. We can express this as $M(R) \sim R^D$ where D is our 'fractal dimension.' (A shape in which D is equal to the dimension of the space the shape is embedded in, the shape is said to be *compact*, for example a cube in there-dimensional space R^3.)

It can be seen that for any fractal object (of size P, made up of smaller units of size p), the number of units (N) that fits into the larger object is equal to the size

---

[5] I am aware this is not technically a word but feel its usage was justified in this case.

[6] Here the term 'mass' is used in lieu of volume so it can be used generally irrespective of dimension.

ratio (P/p) raised to the power of D. We have $N = \left(P/p\right)^D$ or $D = \dfrac{\log N}{\log(P/p)}$ or

$D = \log_r N$ where $r = P/p$, the *aspect ratio* of the shape.

In the above examples this gives:

Shape	N	r	D= $\dfrac{\log N}{\log r}$
Sierpinski Gasket	3	2	1.58
Koch Curve	4	3	1.26
3D Sierpinski Gasket	5	2	2.32
Menger Sponge	20	3	2.09

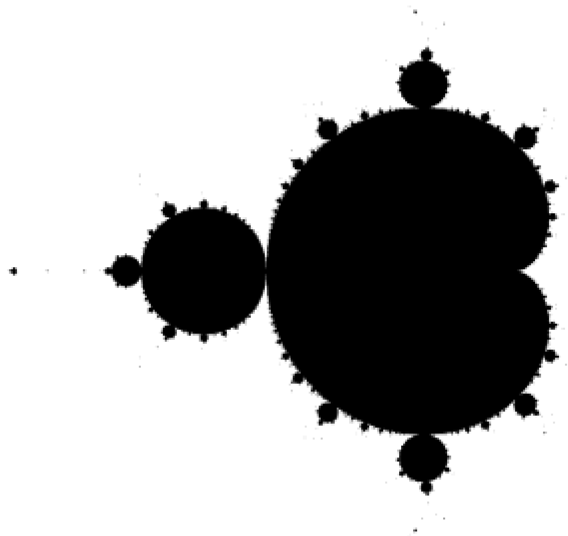
Note here that the values given are vaguely intuitive: The Koch curve seems slightly more than one-dimensional but not enough to be considered two-dimensional, the Sierpinski Gasket seems 'more two-dimensional' than a Koch curve, and then with the 3D Gasket and the Sponge we're heading towards three-dimensionality[7].

9. *Introducing the Mandelbrot Set*

I am now going to take us away from the well-defined Axiom-and-Generator method of defining self-similar fractals and look at more complex objects. Recall the Mandelbrot set described in the introduction:

$$z_0=z \; ; \; z_{n+1} = z_n^2 + z$$

With M the set {z ∈ C such that z_n does not diverge to infinity}, called the Mandelbrot set. As a subset of the complex plane, M can be seen visually as



The above shape is just about contained in the disc of radius two, the origin lies somewhere in the centre of the shape. Note that this shape is, despite appearances, connected – the 'dots' that seem separate from the main shape are connected, but the connection is perhaps too small to see here – we shall

[7] See Footnote Five.

discover this later, on our Mandelbrot Walk. It's also interesting to note that this shape is closed – see Priestley's *Introduction to Complex Analysis*, Exercise 3.16.

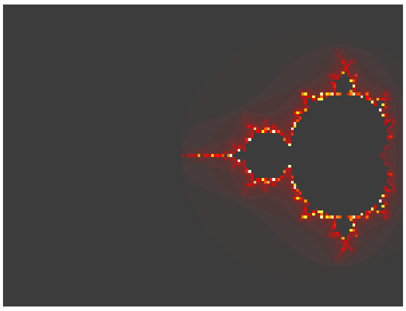
This shape seems rather odd and strange for such a simply defined region. What makes things even more mind-boggling is what happens if one zooms in on the border of the said region: the border gets no less complicated the further down one goes, with (some slightly distorted) versions of the original shape budding off of the main shape – the fact that its not entirely replicating and similar but rather slightly different versions of the original shape makes it a very interesting object indeed. Fractals of this kind become even more interesting when colour is added – the set is coloured black as above, however if we add colour it helps to a) highlight parts of the set that are difficult alone to see in the diagram and b) make Everything Look Pretty. This is how we shall define the colour of a point in a Mandelbrot-style fractal:

Colour(z) = f(0) if $z \in M$ or $f \bullet r(z)$ if $z \notin M$ where r(z) is the number of iterations for a point z to go out of the circle radius 2[8] (this is a footnote, not an exponent,) and f is an arbitrary mapping from integers to colours. (Formally, here r(z) = the least m such that $|z_m|>2$.)

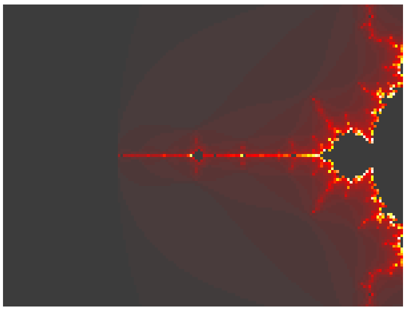
I shall explain how such a mapping f is to work shortly, but firstly let's use this added idea of colour to explore the said Mandelbrot set. Below is a series of images exploring the Mandelbrot set, each image zooming in on the centre of the preceding image…

10. A Mandelbrot Walk

We start off with the Mandelbrot above, with a colour function added:

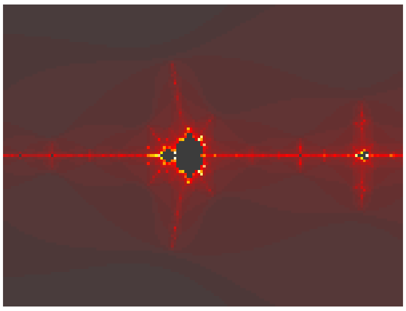


If we know zoom onto the centre of this image, we see the edge of what appears to be this set:

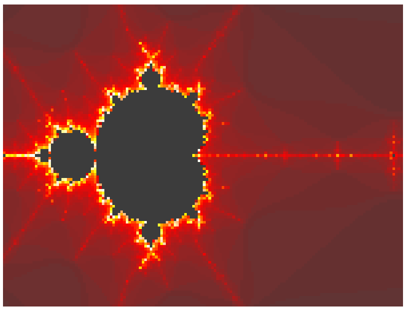


Hmm it appears to have a 'trail' coming off of the fractal to the left if we zoom in on this we see a further bulb:

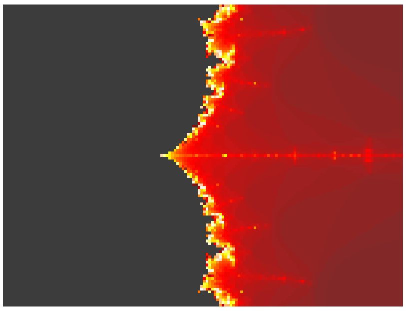
[8] There is a reason for this value: and it shall be revealed. In due time.



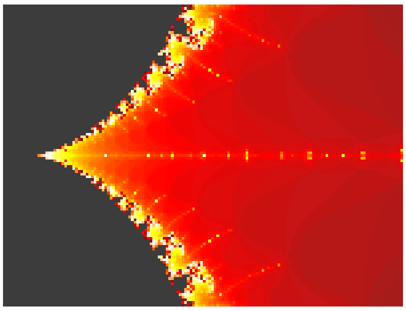
Now this isn't a separate bulb to the first one, it is connected – down the red strip in the centre there is a 'strip' that lies entirely within the Mandelbrot set. Zooming in on this bulb:



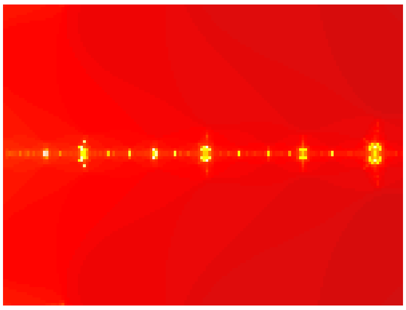
Here we have something that is similar, but not quite the same, as the original set. Zooming in on the centre of this image we start to see the right edge of this bulb:



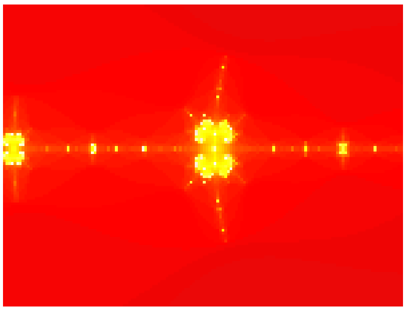
Zooming in on the tip of this we see:



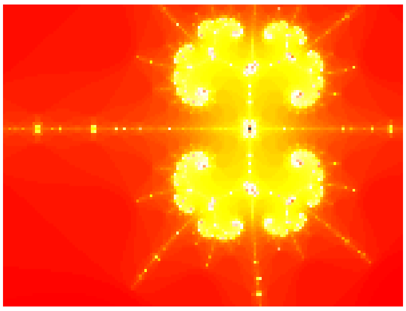
Note here new colours are coming into play that have not been seen in the set so far, again, zooming in further we have



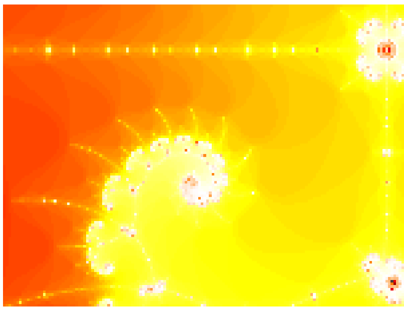
and then

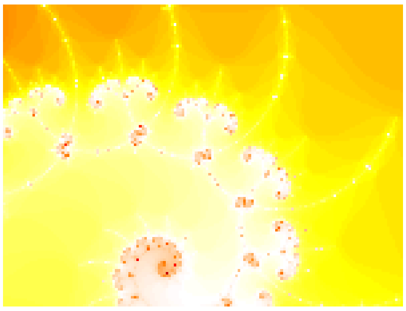


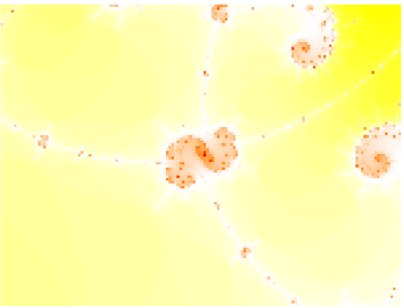
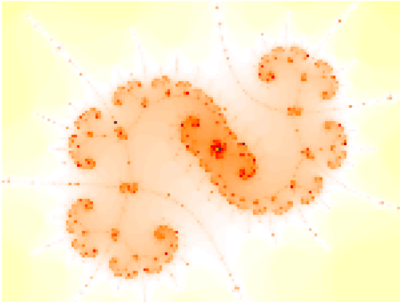
Ooh there looks like there's something interesting (and, indeed, yellow) in the middle there: let's have a closer look.



Hmm that's pretty – didn't see that coming at all. Note that although this is all mostly non-black and not part of the set the set extends to all parts of the yellow here. Zooming further we see the following images:





We could continue this (literally) forever: the deeper we go the more interesting things we find, an infinite amount of an infinitely complex shape. The other beautiful thing about this is how easy it is to create: as its very simple to mathematically define, one can write their own (relatively simple) program to create all of the above imagery – this images having been directly deduced from mathematics itself, thus this amazingly complex image being found in the very nature of logic itself – a strange thought. I have created a said program, which provides most fractal images of this type in this document, and I will discuss in more detail later.

11. The Julia Set(s)

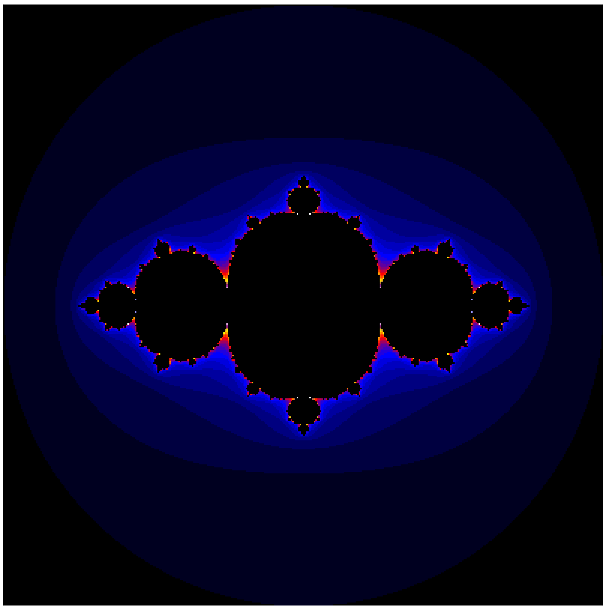
The Mandelbrot can be defined simply as the function given above. An extension of the Mandelbrot is the Julia set. The Mandelbrot set was defined

$$z_0=z \; ; \; z_{n+1} = z_n^2 + z$$

With M the set {$z \in C$ such that z_n does not diverge to infinity} and is similar to a Julia set. A Julia set has a complex (constant) parameter c with

$$z_0=z \; ; \; z_{n+1} = z_n^2 + c$$

With J(c) the set {$z \in C$ such that z_n does not diverge to infinity} For $c=-¾$ we have



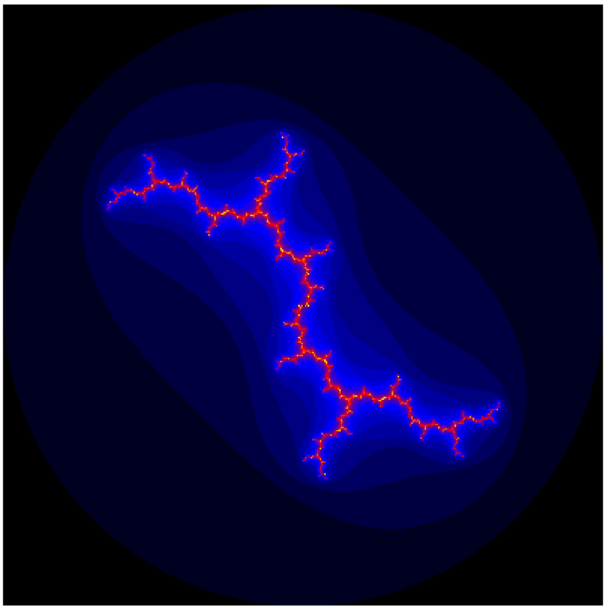
This is interesting: it's a kind of double Mandelbrot-set.

It is also an interesting note that the Julia set and Mandelbrot sets are closely related. The following facts shall now be revealed (although it should be noted that why this facts are the case is beyond the scope of this essay.):

- If c is inside the Mandelbrot set, then the Julia set for c will be connected. If c is outside the Mandelbrot set, then the Julia set for c will be disconnected (that is, it will have at least two disconnected islands.)
- If we choose a c from inside the Mandelbrot Set, its location inside the set will have a radical influence in the shape of the respective Julia set. The closer c is to the border of the Mandelbrot Set, the thinner and whirlier the corresponding Julia set will be. If we choose a c further from the border, the respective Julia set will be thicker.
- If we choose a c that is very close to the border of the Mandelbrot set, there will also be a close relation between the shape of the Julia set and the shape of the border of the Mandelbrot around said c – i.e. zooming in the Mandelbrot set around c will bring up shapes that look like the Julia set for c.

For these reasons it is said that the Mandelbrot set is a "map" of all the Julia sets. If one were to have the time, one could create a computer program that shows an image of the Mandelbrot set and next to it an image of the Julia set - with c set to where the cursor is on the Mandelbrot set. This would demonstrate their relationship rather nicely.

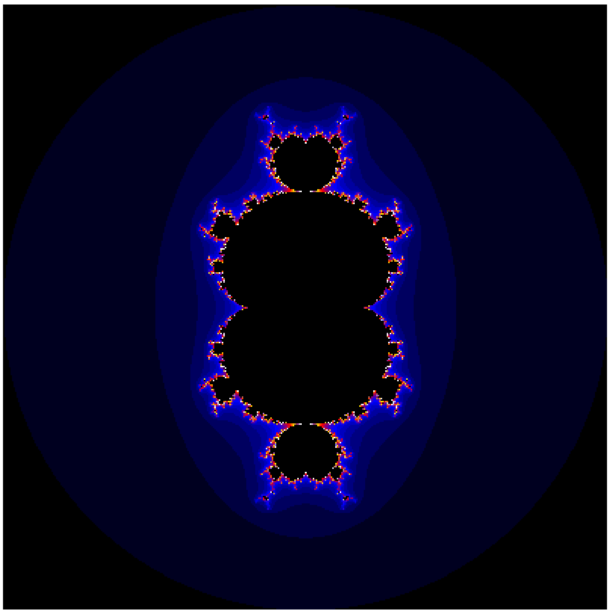
If we set c as the complex value –i we get the following image



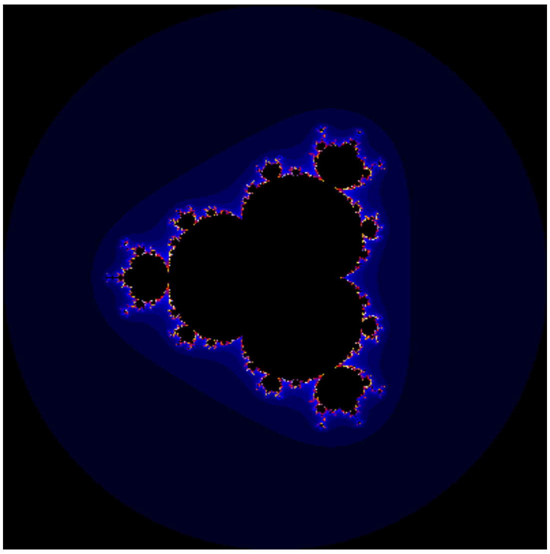
This is interesting as it is known that –i lies on the border of the Mandelbrot set: so this fits in with the deductions above. (This particular shape, like the Sierpinski Gasket, has a zero area and an infinite perimeter.) I find the complex nature of these shapes, and how the Mandelbrot set maps the Julia set, very interesting indeed, especially considering how simple their definitions are.

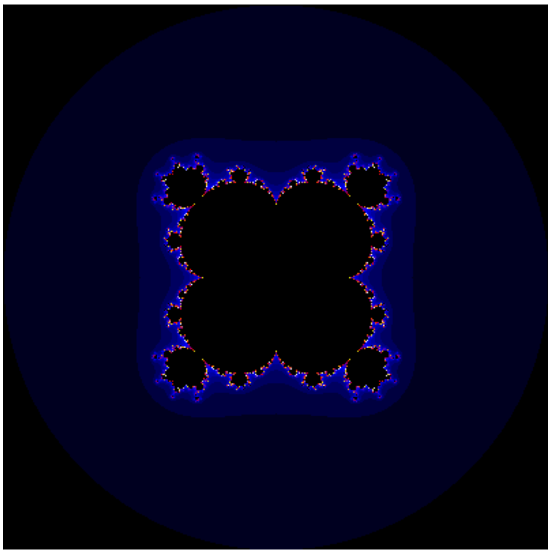
12. Other Said Fractals

In the Mandelbrot set, we square the value and add z. What happens if we cube it?



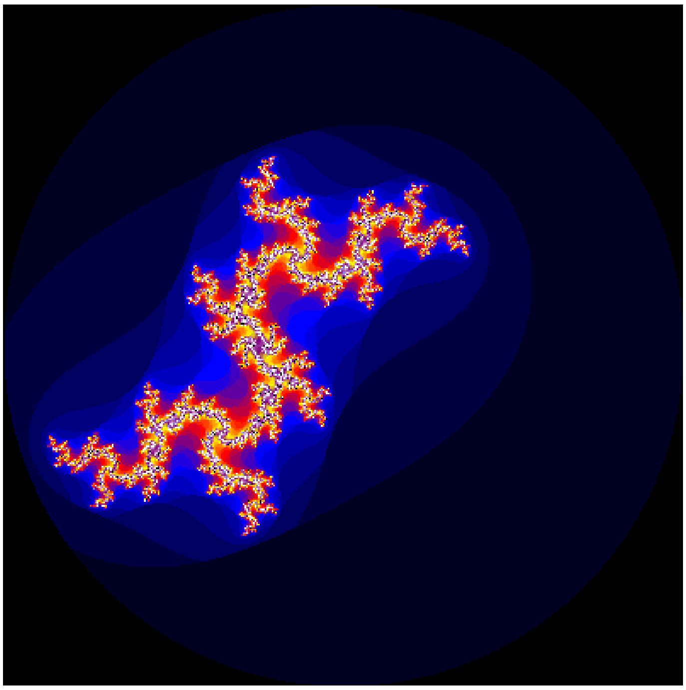
What about the fourth and fifth powers?





Here notice how the shape tends to have n-1 'bulbs' to it – this is quite strange and mindboggling, as the value of n affects the colour at only a pointwise level, so the fact that in can be seen in the general shape I find quite amazing indeed.

This is the Manowar set, which uses $z_n = z_n^2 + z_n + c$ with a c parameter, in this case, of ¼(1-i):



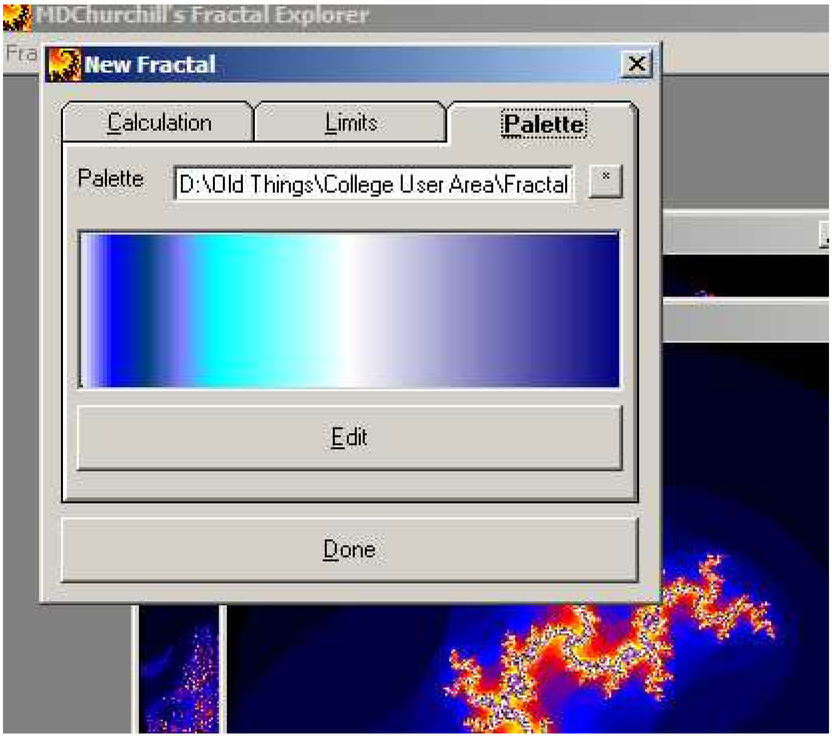
In all of these cases infinitely complex shapes are being shown that have arisen directly from the simple formula given. Here we have an example of chaos theory: very simply defined objects portraying very complex behaviour.

Though I shall not go into this in much detail, the dimension of all of these style of fractals are very close to 2 indeed – they are as wiggly as one can go without being flat. Calculating the Hausdorff dimension of this type of fractal is beyond the scope of this essay.

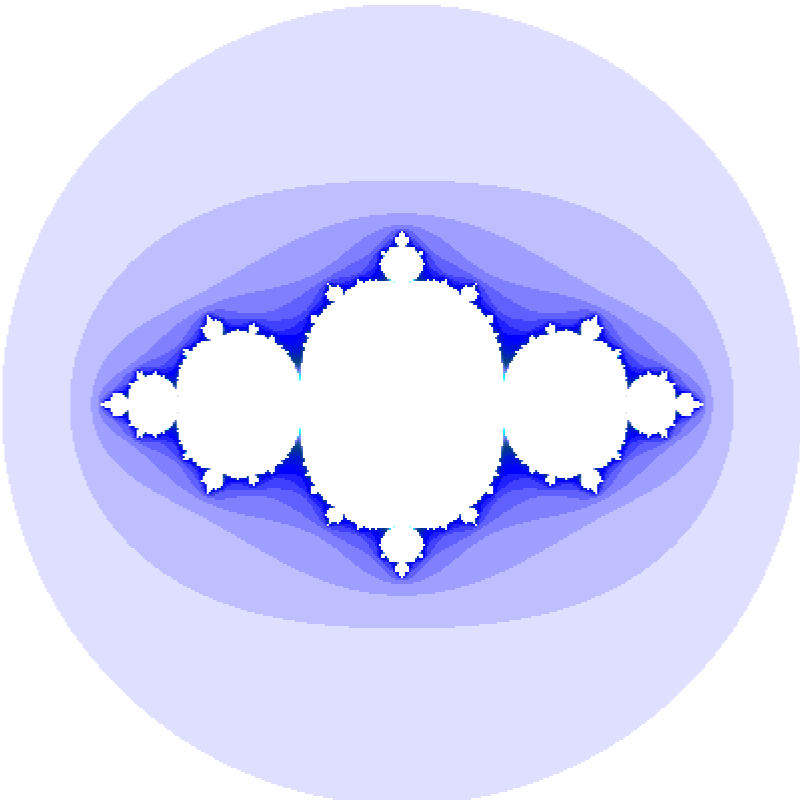
All of the fractals in this section were created by a fractal program written by myself. I shall now explain my colour-generating function and a few other interesting details involved in my creation of said program.

13. Creating the Fractals

Clearly we need a mapping f above from integers to colours. The way I have done this in my program is by using what I referred to as a 'palette' – formally a palette is a subset of $N \times Co$ where N is the set of natural numbers (positive integers) and Co is the set of colours (a colour could be defined as an element of $[0,1] \times [0,1] \times [0,1]$ representing the red, green and blue components respectively.) So we have a set of points and their representing colours, for example {(0,Black),(8,Red),(16,Blue),(32,Yellow),(64,White),(128,Purple)} is the palette for the images used in the preceding section – thus if a particular point required 16 iterations to escape out of D(2) it would be coloured blue. If the value is not available, the program interpolates – for example if it required 20 iterations it would be coloured a slightly yellowy blue, or 12 iterations would give a purply colour. Another example palette is an arctic palette consisting of bluey colours:



this gives a Julia set looking like this:



In addition in creating said fractals, the basic algorithm then would be to loop through each pixel (fundamental particle of colour on the image,) calculate its position on the complex plane, keep looping the sequence until the sequence tends to infinity, and colour the particle depending on whether it does and, if it does, how long it takes. However, there is something fundamentally impossible about the previous sentence, and therefore an extra level of complication is required: Clearly it is impossible for a computer to test whether a given sequence of numbers tends to infinity just by looking at these numbers – or rather, the computer would take an infinite time to do so (due to the definition of a sequence tending to infinity.) Fortunately however, the following fact can now be revealed:

- If in these fractals the value of $|z_n|$ exceeds two (2), it will never return within this circle.[9]

This condition is explored with the Mandelbrot set in Priestley, Introduction to Complex Analysis, Exercise 3.16. Thus, we only need to test until the value exceeds 2. So we can definitely tell if a complex number z is *not* in the set, however, there is still a fundamental problem: we cannot test 'for ever' to check the sequence 'always' remains in D(0,2) (the disc centre the origin radius 2.) Thus we set some 'maximum tries' value M and if $|z_n| < 2 \ \forall n \le M$ then we assume (z_n) does not tend to infinity. This then doesn't give us the exact shape, however a high enough M will make it accurate enough given the given resolution, so a function could be used which takes the resolution R and from it calculates a minimal value of M which would give the exact shape given R.

So the above algorithm is refined to

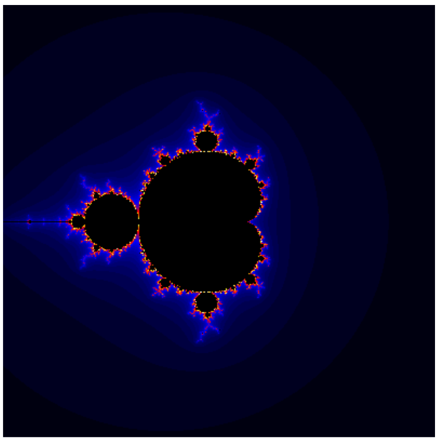
```
For each pixel p on the screen
      Calculate position on complex plane
      While zn < 2 or n > M
            Increment n by one
      Colour the pixel p f(n) where f is the palette interpolation
      function.
```

Note that this algorithm has complexity $O(M\times|p|)$ where $|p|$ is the number of pixels. This is obviously quite large, explaining the length of time required to calculate the fractals.

14. Hybrid fractals

When I was experimenting with fractals of this variety, I noticed something remarkably remarkable. It is not dissimilar to the interesting observation earlier regarding the number of bulbs on the N-Mandelbrot sets. After a particularly inspired dog-walk, I found myself reasoning thus: What if we were to try to 'mix' two fractals on a fundamental level? Would the resulting image seem some kind of mixture of the two original images?

Here we have the Mandelbrot set:



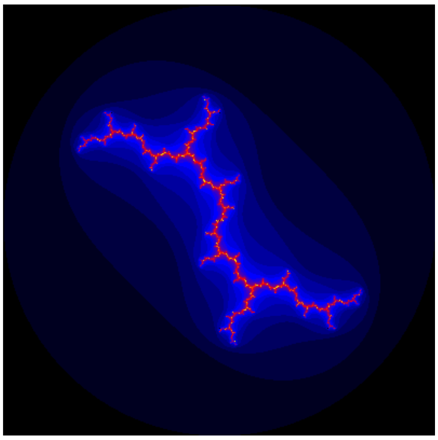
[9] This is the said reason as to why the fractals are coloured in the way described on previous pages.

Defined

$$z_0=z\ ;\ z_{n+1} = z_n^2 + z$$

And the Julia set with c=-i

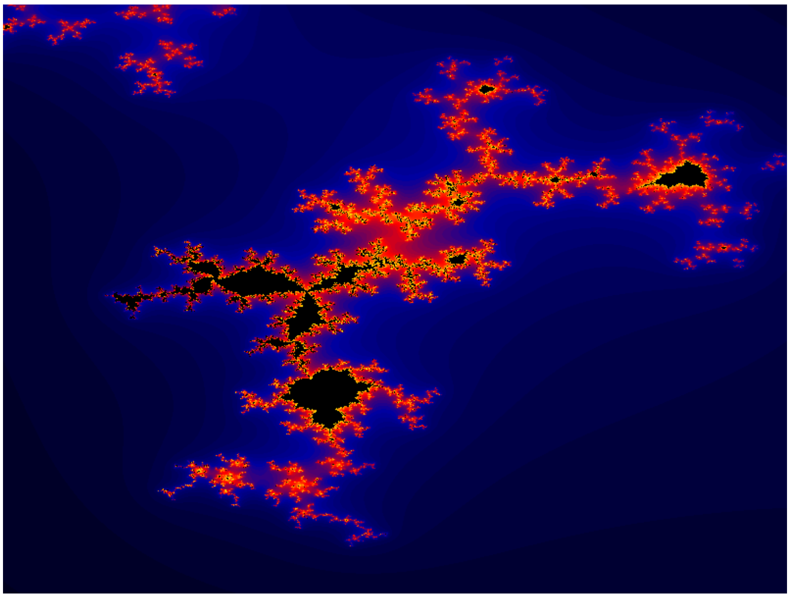
$$z_0=z\ ;\ z_{n+1} = z_n^2 - i$$



Behold the following sequence z_n

$$z_0=z\ ;\ z_{n+1} = z_n^2 - i \text{ if n is even, } z_{n+1} = z_n^2 + z \text{ if n is odd.}$$

Curiously, we get the following shape:



I find this very intriguing indeed: we combine the two formulae on a fundamental pointwise level and the image we get is something that looks like a mixture of the two fractals – a Julia shape with little bits of Mandelbrot in there. On exploring the idea, any time I "crossed" to fractals in the given way, the final image came out as a strange mixture of the two original fractals, with bits of each attached to a strange hyperfractal.

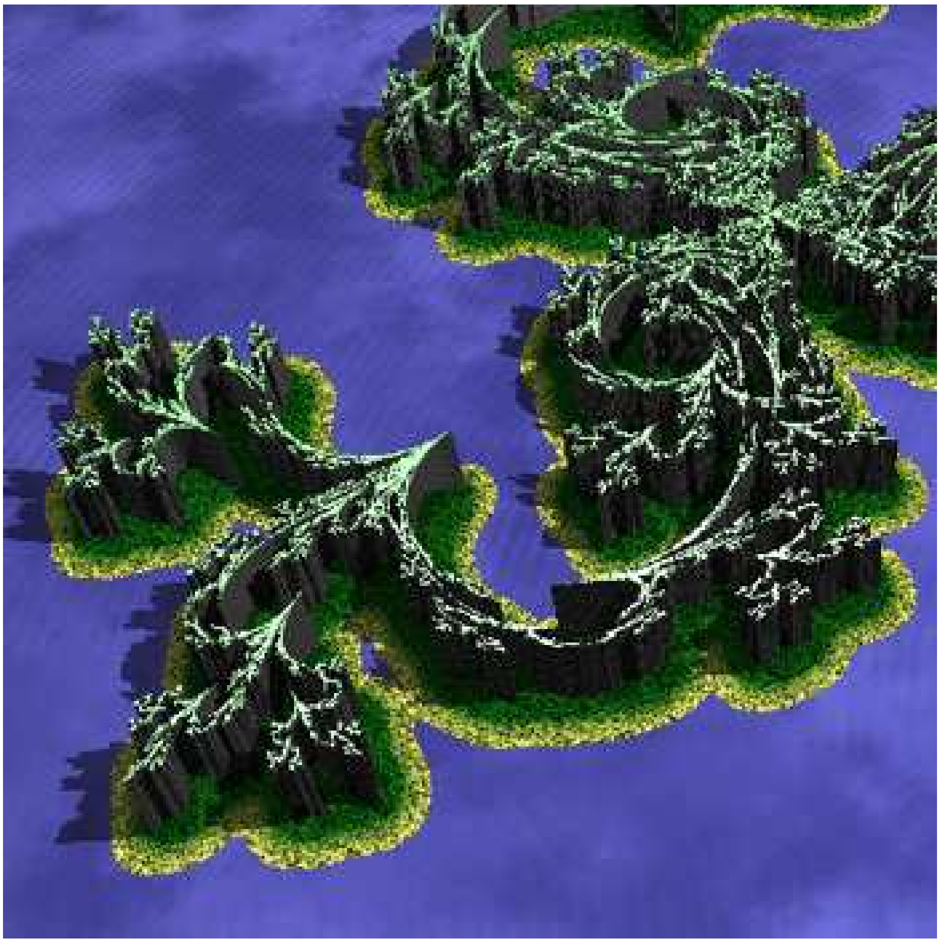
I find this interesting, and it may, in some abstract way, relate to the previous exploration of the relationship between the Mandelbrot and Julia sets. However, on exploration I find that this strange phenomenon occurs also in other non-Julia/Mandelbrot fractals. I have dubbed this concept 'hybrid fractals' and haven't seemed to be able to find any information on them on the internet, however, this doesn't mean they're not there. I imagine this idea has probably been explored before, though of course it may not have been…

15. Aliens, Islands and Cacti

Behold the following fractal:

If we blur the boundary slightly, we can do something a bit special to it. Firstly if we add an extra dimension to it

Then we can render it and do this:

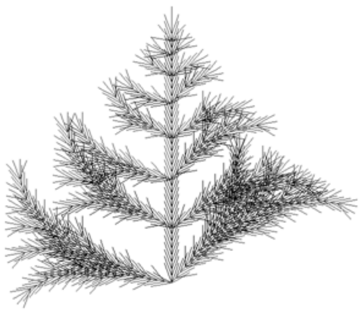


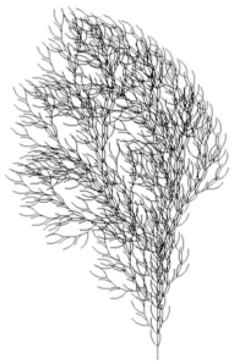
And we have our Fractal Island. This type of method is used a reasonable in computer graphics, able to create weird and wacky things such as alien landscapes. The above example is simple, but as the complexity of the fractal increases we get images such as the following:

This fractal, of the 'self-similar' variety described earlier, looks remarkably tree like. The following extract is from an article in *Interface Magazine, 1990*, and it describes the idea rather nicely:

"Computer graphics using CAD software is typically good at creating representations of man-made objects using primitives such as lines, rectangles, polygons, and curves in 2 D or boxes and surfaces in 3D. These geometric primitives and usual tools for manipulating them typically prove inadequate when it comes to representing most objects found in nature such as clouds, trees, veins, waves, and a clump of mud. There has been considerable interest recently in chaos theory and fractal geometry as we find that many processes in the world can be accurately described using that theory. The computer graphics industry is rapidly incorporating these techniques to generate stunningly beautiful images as well as realistic natural looking structures."

It takes just a few bytes of data to store the code to generate the following pattern:



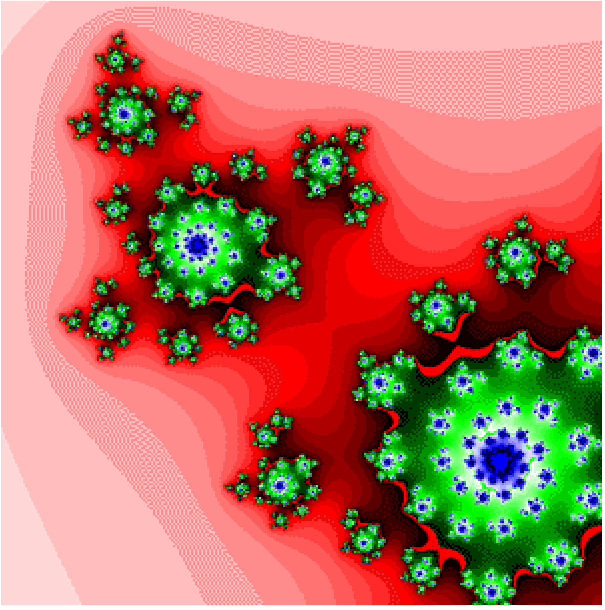


And it seems to make a lot more sense for computer graphics designers to store the relatively small amount of data it takes to create these images than the images themselves.

The following function

$$z_{n+1} = \sin(z_n^2) + c$$

Gives rise to a biological image:

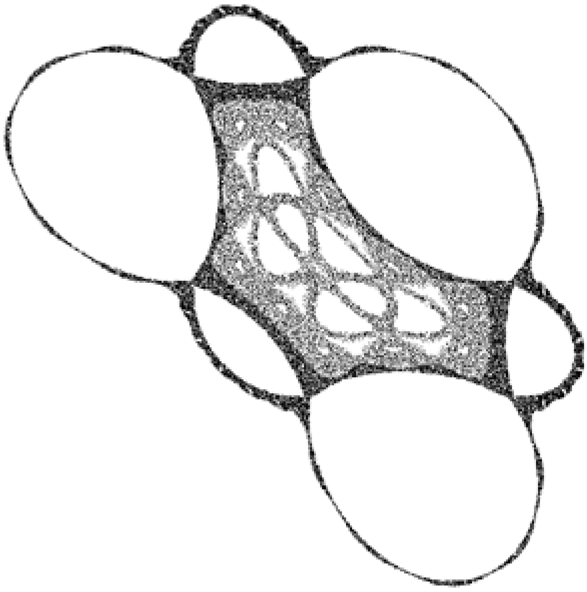


In which, with a bit of imagination, we have a pool of alien life…

And

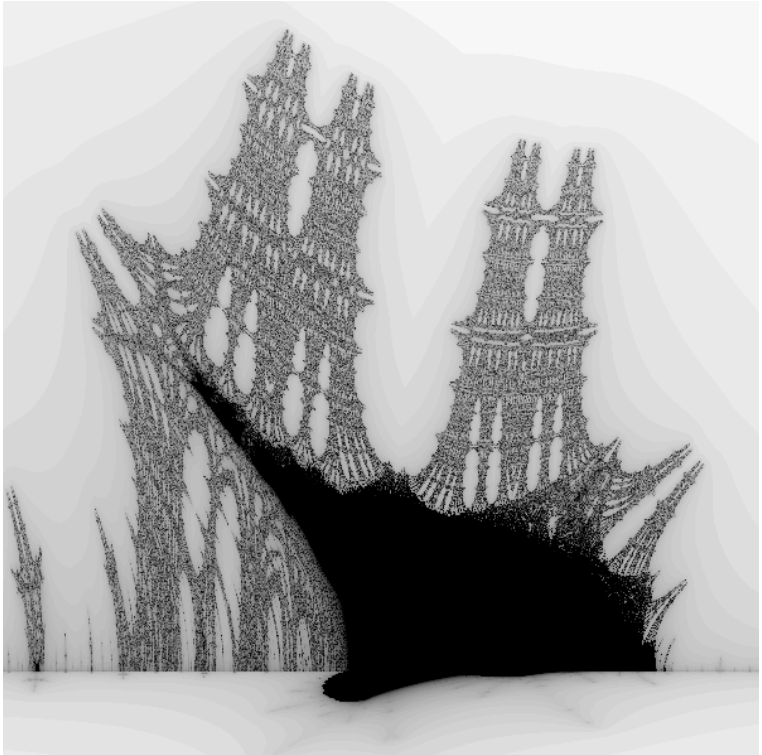
$$z_{n+1} = u\ z_n(1 - z_n).$$

With u = -0.7 + 0.8I gives us a galaxy for them to live:

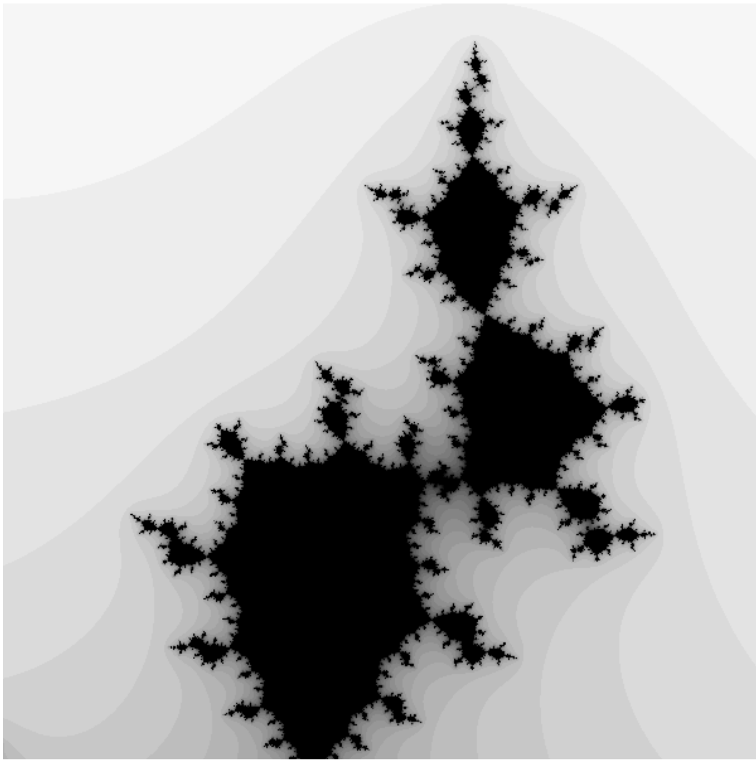


This certainly paves the way for a new art form, discovering and developing images based on the formulas that create them.

Here we have a burning ship:



And, of course, this section would not be complete without the obligatory cactus, as so teasingly promised in the title of this chapter:



16. Conclusion

This hereby concludes are trip through Fractaldom. I have explored the concept of fractals, infinitely complex shapes generated from simple patterns and formulae (comparisons to Chaos theory here are very welcome.) I have largely looked at a vague outline of what may be going on: it certainly would be an interesting endeavour to explore some of these ideas in far more rigour. We have looked at quite a simple well-defined way of doing this through self-similarity, then moved onto more complex types of fractals that are nonetheless straightforward to define. Briefly, I have discussed how I implemented creating said fractals, and concluded with a few 'special' topics such as looking at hybrid fractals and cacti, the later case showing how the idea of fractal images are finding there way into computer graphics (fundamentally, the way interesting images can be created in reasonably simple ways.) I believe this gives a nice way showing how some of these fractals are being used 'in the real world' (lit: outside mathematical curiosity) and therefore perhaps gives a nice way to conclude this outlining exploration of some of the ideas within fractal geometry.